Efficient learning and simulation of quantum continuous variable systems

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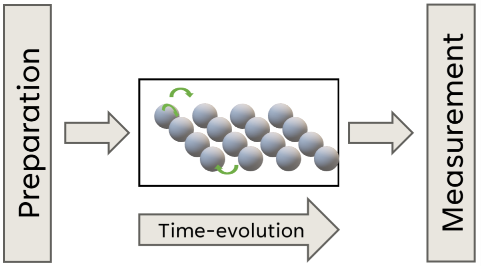
Introduction



Hamiltonian learning: The task of identifying the unknown Hamiltonian governing the evolution of a quantum system.

Learning bosonic systems

- Most work has focused on finite-dimensional quantum systems, e.g., collections of qubits.
- However, bosonic systems are not finite-dimensional and are described using unbounded operators, which is mathematically challenging.
- Examples: superconducting circuits, integrated photonic circuits, optomechanical platforms
- Aim: Efficient learning, i.e., evolution time scales as $\mathcal{O}(\varepsilon^{-1})$ (Heisenberg limit).
- Previous works are either restricted to Bose-Hubbard-like models [LTGNY24] or do not achieve Heisenberg scaling [MBCWR23].
- This work combines the best of both worlds.

Low-intersection bosonic Hamiltonians

Annihilation and creation operators: $b_i \left|k\right\rangle_i = \sqrt{k} \left|k-1\right\rangle_i$, $b_i^\dagger \left|k\right\rangle_i = \sqrt{k+1} \left|k+1\right\rangle_i$.

Definition

A low-intersection bosonic Hamiltonian acting on m modes is a Hamiltonian that takes the form $H = \sum_{a=1}^{M} E_a$, where each E_a is an $\mathfrak{k}$-mode interaction of the form

$$E_a = \sum_{\mathbf{j},\mathbf{j}' \in \mathbb{N}^{\mathfrak{k}} : \|\mathbf{j}+\mathbf{j}'\|_1 \le d} h^{(a)}_{\mathbf{j},\mathbf{j}'} (\mathbf{b}^\dagger)^{\mathbf{j}} \, \mathbf{b}^{\mathbf{j}'} .$$

We assume that at least one of $\mathbf{j} \neq 0$ or $\mathbf{j}' \neq 0$ holds and assume that $|h^{(a)}_{\mathbf{j},\mathbf{j}'}| \le 1$. Moreover, $\mathfrak{k} = \mathcal{O}(1)$ and each E_a overlaps with at most $\mathfrak{d} = \mathcal{O}(1)$ other interactions E_b.

Our learning protocol generates estimates $\hat{h}^{(a)}_{\mathbf{j},\mathbf{j}'}$ such that

$$\max_{\mathbf{j},\mathbf{j}'} |\hat{h}^{(a)}_{\mathbf{j},\mathbf{j}'} - h^{(a)}_{\mathbf{j},\mathbf{j}'}| \le \epsilon \text{ with probability at least } 1-\delta.$$

Heisenberg-limited learning of low-intersection bosonic Hamiltonians

For our learning algorithm, we will make use of engineered dissipation as in [MBCWR23]. Recall: m modes, Hamiltonian of degree d

Theorem

There exists an algorithm which makes use of dissipation with strength $\gamma = \mathcal{O}(m^2 \varepsilon^{-1} \log^{2d+1/2}(1/\varepsilon))$ that can estimate all coefficients of H to precision ϵ with probability at least $1-\delta$. It requires

$$\begin{aligned} \mathcal{O}((1/\varepsilon)\log(m/\delta)) & \quad \textit{total evolution time, and} \\ \mathcal{O}(\log^2(\log(1/\varepsilon)/\varepsilon)\log(m/\delta)) & \quad \textit{experiments.} \end{aligned}$$

Core insight: By adding sufficiently strong engineered dissipation, we can restrict the time evolution to a subspace of our choosing, which allows us to extract the coefficients of H from the time evolution under an effective Hamiltonian.

Added dissipation

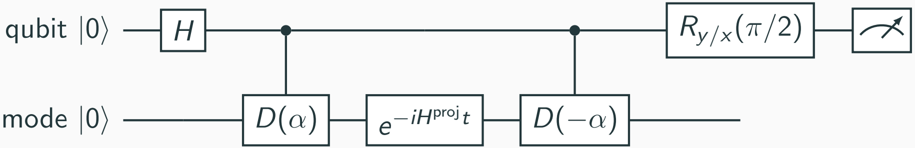
- The time evolution with added dissipation is defined by
$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \gamma\mathcal{L}(\rho(t)), \qquad \rho(0) = \rho_0 \,.$$
- The generator $\mathcal{L}$ of the dissipation consists of single-mode terms $\mathcal{L}_i$. On each mode i, for $\alpha_i \in \mathbb{C}$ we consider a dissipation in the Lindbladian form $\mathcal{L}_i := \mathcal{L}[L_{1,\alpha_i}] + \mathcal{L}[L_{r_i,\alpha_i}]$, where $\mathcal{L}[L] = L \cdot L^\dagger - \frac{1}{2}\{L^\dagger L, \cdot\}$.
- The jump operators of the Lindblad operators $\mathcal{L}[L_{r,\alpha}]$ are
$$L_{r,\alpha} = b^r(b - \alpha)\,.$$
Here, r_i has to be chosen sufficiently large as a function of d.
- The first part of the generator projects on the correct subspace, the second one controls the Hamiltonian.
- One can use Trotterization to alternate between Hamiltonian evolution and dissipation for short times.

Algorithm (1/2)

- Idea: The dissipative time evolution is approximately equal to the unitary time evolution on span$\{|0\rangle, |\alpha\rangle\}$ generated by an effective Hamiltonian which approximately has the form

$$H^{\mathrm{proj}} \approx \begin{pmatrix} 0 & 0 \\ 0 & \langle\alpha| H |\alpha\rangle \end{pmatrix}.$$

- For different values of α and times $\{t_i\}_i$, we run the following circuit, where $D(\alpha)$ is the displacement operator:



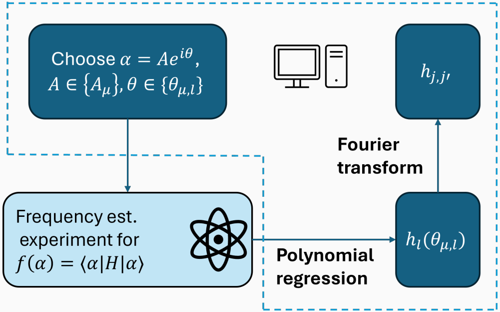
- These experiments estimate well $\langle X \otimes I\rangle_{\rho_\alpha(t_i)}$ and $\langle Y \otimes I\rangle_{\rho_\alpha(t_i)}$.

Algorithm (2/2)

- We can use a phase estimation algorithm to estimate $\langle\alpha| H |\alpha\rangle$ as

$$\langle X \otimes I\rangle_{\rho_\alpha(t_i)} \approx \cos(\langle\alpha| H |\alpha\rangle\, t_i)\,,$$
$$\langle Y \otimes I\rangle_{\rho_\alpha(t_i)} \approx \sin(\langle\alpha| H |\alpha\rangle\, t_i)\,.$$

- Classical post-processing: Writing $\alpha = Ae^{i\theta}$, we find that $\langle\alpha| H |\alpha\rangle$ is a polynomial in A. Thus, we use Chebyshev interpolation to estimate its coefficients and extract the $\hat{h}_{j,j'}$ from them via a Fourier transform.



Adiabatic approximation for general Lindbladian evolutions

Our main technical contribution is the following theorem:

Theorem

Let H be a self-adjoint operator defined on a Hilbert space $\mathscr{H}$, and let $\mathcal{L}$ be a Lindbladian over the state space on $\mathscr{H}$. Let P be the orthogonal projection onto the intersection of the finite-dimensional kernels of the jump operators defining $\mathcal{L}$. Then, under some reasonable technical conditions on H and $\mathcal{L}$, for any state $\rho = P\rho P$ and $\gamma > 0$,

$$\left\| e^{t(\gamma\mathcal{L}+\mathcal{H})}(\rho) - e^{t\mathcal{H}_{\text{proj}}}(\rho) \right\|_1 \leq \frac{tC}{\gamma} + \frac{C'}{\gamma},$$

where C and C' are constants. Here, $\mathcal{H} = -i[H, \cdot]$ and $\mathcal{H}_{\text{proj}}$ is an effective version of $\mathcal{H}$ restricted to the image of P.

If the dissipation strength γ is large enough, the time evolution generated by H is restricted to the invariant subspace of the dissipative evolution generated by $\mathcal{L}$.

Application to cat codes (1/2)

- Bosonic cat codes rely on r-photon drive dissipation:

$$\frac{d}{dt}\rho(t) = \mathcal{L}[L_r](\rho(t)) \qquad \text{with} \qquad L_r := b^r - \alpha^r\,.$$

- For large times, the evolution drives any state exponentially fast to the finite-dimensional code space

$$\mathcal{C}_r(\alpha) := \mathrm{span}\left\{|\alpha_1\rangle\langle\alpha_2| : \alpha_1, \alpha_2 \in \left\{\alpha e^{\frac{i2\pi j}{r}} : j \in \{0, \dots, r-1\}\right\}\right\}$$

as (see [ASR16])

$$\mathrm{tr}\left[L_r e^{t\mathcal{L}[L_r]}(\rho) L_r^\dagger\right] \leq e^{-tr!}\,\mathrm{tr}\left[L_r \rho L_r^\dagger\right].$$

Application to cat codes (2/2)

Using our adiabatic approximation theorem, we can prove convergence of the quantum dissipative evolution in the natural trace norm. Moreover:

Theorem

Let H a single-mode low-intersection bosonic Hamiltonian, $d/2 \leq r$, and let P be the orthogonal projection onto $\mathcal{C}_r(\alpha)$. Then, for all $t \geq 0$ and $\rho \in \mathcal{C}_r(\alpha)$,

$$\| e^{-it[H,\cdot]+t\gamma\mathcal{L}[b^r-\alpha^r]}(\rho) - e^{-it[PHP,P\cdot P]}(\rho)\|_1 \leq \frac{tC}{\gamma} + \frac{C'}{\gamma}$$

for constants $C, C' \geq 0$.

- The r-photon driven dissipation of sufficient strength leads to an effective time evolution on the code space for any bounded-degree bosonic Hamiltonian.
- For $r = 2$, this specifically provides explicit convergence bounds for implementing rotations around the x-axis on the code space, experimentally realized [TGL+18].

Summary

- We have proved a new adiabatic approximation for dissipative evolutions
- Leads to a new Hamiltonian learning algorithm with Heisenberg scaling for low-intersection bounded degree systems
- Idea: Use dissipation to restrict evolution to a subspace on which we can learn
- Another application: Exponential convergence of photon driven dissipation to the cat code space

Some open questions:

- What is the optimal strength γ required to confine the effective evolutions within the finite-dimensional subspace? Can we get a scaling independent of m?
- How can the required dissipation be realized in an experiment?

References

[ASR16]: R. Azouit, A. Sarlette, and P. Rouchon. Well-posedness and convergence of the Lindblad master equation for a quantum harm. oscillator with multi-photon drive and damping. *ESAIM: Control, Optimisation and Calculus of Variations*, 22(4), 2016.

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[LTGNY24]: H. Li, Y. Tong, T. Gefen, H. Ni, and L. Ying. Heisenberg-limited Hamiltonian learn. for interacting bosons. *npj Quantum Information*, 10(1):83, 2024.

[TGL+18]: S. Touzard, A. Grimm, Z. Leghtas, *et al.* Coherent oscillations inside a quantum manifold stabilized by dissipation. *Physical Review X*, 8(2), 2018.

Our paper: